Zero-conductance resonances and spin filtering effects in ring conductors subject to Rashba coupling

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We investigate the effect of Rashba spin-orbit coupling and of a tunnel barrier on the zero conductance resonances appearing in a one-dimensional conducting Aharonov-Bohm (AB) ring symmetrically coupled to two leads. The transmission function of the corresponding one-electron problem is derived within the scattering matrix approach and analyzed in the complex energy plane with focus on the role of the tunnel barrier strength on the zero-pole structure characteristic of transmission (anti)resonances. The lifting of the real conductance zeros is related to the breaking of the spin-reversal symmetry and time-reversal symmetry of Aharonov-Casher (AC)and AB rings, as well as to rotational symmetry breaking in presence of a tunnel barrier. We show that the polarization direction of transmitted electrons can be controlled via the tunnel barrier strength and discuss a novel spin-filtering design in one-dimensional rings with tunable spin-orbit interaction.

I. INTRODUCTION

In the last decade enormous attention, from both experimental and theoretical physics communities, has been devoted towards control and engineering of the spin degree of freedom at the mesoscopic scale, usually referred to as spintronics.[1,2] The major goal in this field is the generation of spin-polarized currents and their appropriate manipulation in a controllable environment, preferably in semiconductor systems. Since the original proposal of the spin field effect transistor (spin FET) by Datta and Das[3], many proposals have appeared, but the realization of a spin transistor or spin filter still remains challenging. Of particular interest appear setups based on intrinsic spin dependent properties of semiconductors, like the Rashba spin orbit (SO) effect[5,6] for a two-dimensional electron gas confined to an asymmetric potential well. This is a dominant mechanism for the spin-splitting in semiconductors that has been proven to be a convenient means of all-electrical control of spin polarized current through additional gate voltages[7]. In addition, suitable means for controlling spin at mesoscopic scales are provided by quantum interference effects in coherent ring conductors under the influence of electromagnetic potentials, known as Aharonov-Bohm[8] and Aharonov-Casher[9] effect. This possibility has driven a wide interest in spin-dependent Aharonov-Bohm physics, and the transmission properties of mesoscopic AB and AC rings coupled to current leads have been studied under various aspects such as AB flux and coupling dependence of resonances[10,11], Berry phases[12,13,14,15,16,17], spin-related conductance modulation[18,19,20], persistent currents[13,21], spin filters[22,23] and detectors[24], spin rotation[25], and spin switching mechanisms[26].

In this paper we focus on the zero conductance resonances in a one-dimensional AB ring subject to Rashba spin-orbit interaction[27] and interrupted by a tunnel barrier in the lower arm. First, motivated by the work of Aeberhard et al.[11], we revisit the subject of spin-induced modulation and of zero-pole structure of the conductance of a symmetrically coupled ring as a function of the Rashba coupling strength and extract distinct effects due to the presence of tunnel barrier in one of the arms which have not been considered in earlier works. Using the effective Hamiltonian for one-dimensional (1D) rings, as recently considered in Refs.[19,29] and taking into account the corresponding appropriate eigenstates, in Sec.II we derive the analytic expression of the conductance in presence of AB-AC fluxes and of the tunnel barrier. Later on, we discuss the real zeros conductance and the zeros lifting due to the SO interaction and the tunnel barrier strength. The imprints of the presence of a tunable tunnel barrier together with the Rashba coupling (strength) on the overall conductance is remarkable. In fact, in Sec.III we demonstrate, by means of numerical and analytical calculations, that the interference zeros of the spin-resolved conductance in one spin-channel can be compensated by poles, while the location of zeros due to the presence of the tunnel barrier can be controlled by means of its strength. This implies a spin-filter mechanism which is probably more convenient for experimental realizations than previous proposals[22,26,28], due to presence of intrinsic effective field associated to the Rashba interaction and to a highly controllable parameter, the tunnel barrier strength. We present a short summary in Sec.IV.

II. THE AB RING WITH TUNABLE RASHBA SO COUPLING

A. Hamiltonian and the one-particle solution

As known, spin-orbit coupling (SOC) is due to a magnetic field generated in the reference frame of the moving electron by an electric field in the reference frame of the laboratory. When considering a one dimensional ring in a semiconductor structure, an effective Rashba electric field results from the asymmetric confinement along the direction (k) perpendicular to the plane of the ring[5,6]. A consequence of lack of inversion symmetry in presence of a confinement potential $V(k)$ is a spin band splitting proportional to the momentum of the electron. The Hamiltonian describing SO coupling is the following:

$$\hat{H}_{SO} = \frac{\alpha}{\hbar}(\hat{\vec{\sigma}} \times \hat{\vec{p}})_k, \tag{1}$$

where $\hbar/2\hat{\vec{\sigma}}$ is the spin operator expressed in terms of the Pauli spin matrices, $\hat{\vec{\sigma}} = (\sigma_i, \sigma_j, \sigma_k)$ and α is the SOC associated to the effective electric field along the k direction. The total Hamiltonian of a moving electron in presence of SOC can be found in Ref.[30]. In the case of a one-dimensional ring an additional confining potential, $V_c(r)$ must be added in order to force the electron wave function to be localized on the ring. A typical confining potential is an harmonic potential centered at the radius of the ring, $V_c(r) = \frac{1}{2}K(r - R)^2$. When only the lowest radial model is taken into account, the resulting effective one dimensional Hamiltonian in a dimensionless form[19,29] can be written as:

$$\hat{H} = \frac{2m^\star R^2}{\hbar^2}\hat{H}_{1D} = \left(-i\frac{\partial}{\partial\varphi} + \frac{\beta}{2}\sigma_r\right)^2, \tag{2}$$

where $m^\star$ is the effective mass of the carrier, $\beta = 2\alpha m^\star/\hbar^2$ is the dimensionless SOC, $\sigma_r = \cos\varphi\sigma_i + \sin\varphi\sigma_j$, and additional constants have been dropped. The parameter α represents the average electric field along the k direction and is assumed to be a tunable quantity. For an InGaAs-based two-dimensional electron gas, α can be controlled by a gate voltage with typical values in the range $(0.5 \div 2.0) \times 10^{-11}$eVm[31,32]. In presence of a finite AB magnetic flux and of a tunnel barrier localized in the lower arm of the ring, the Hamiltonian can be generalized to:

$$\hat{H} = \left(-i\frac{\partial}{\partial\varphi} + \frac{\beta}{2}\sigma_r - \frac{\Phi_{AB}}{\phi_0}\right)^2 + v\delta(\varphi' + \frac{\pi}{2}), \tag{3}$$

where $\phi_0 = hc/e$ is the quantum flux and v is the dimensionless tunnel barrier strength $v = 2m^\star R^2 V/\hbar^2$ which can be tuned by an external gate voltage applied at a quantum point contact[47] and $\varphi' = -\varphi$. As outlined in the Appendix of Ref.[29] one can solve the eigenvalue problem in a straightforward manner and the energy eigenvalues are:

$$E_n^\sigma = (n - \Phi_{AC}^\sigma/2\pi - \Phi_{AB}/2\pi)^2, \tag{4}$$

where $\sigma = \pm$, Φ_{AC}^σ is the so-called Aharonov-Casher phase[9]:

$$\Phi_{AC}^\sigma = -\pi(1 - \sigma\sqrt{\beta^2 + 1}). \tag{5}$$

At fixed energy, the dispersion relation yields the quantum numbers $n_\lambda^\sigma(E) = \lambda\sqrt{E} + \Phi^\sigma/2\pi$, where we have introduced $\Phi^\sigma = \Phi_{AC}^\sigma + \Phi_{AB}$, and the index $\lambda = \pm$ refers to right/left movers, respectively. The eigenvectors have the general form[19,29]:

$$\Psi_n^\sigma(\varphi) = e^{in\varphi}\chi^\sigma(\varphi), \tag{6}$$

where $n \in \mathbb{Z}$ is the orbital quantum number. It should be noted that the spinors $\chi^\sigma(\varphi)$ are generally not aligned with the Rashba electric field, but they form a tilt angle given by $\tan\theta = -\beta$ relative to the k direction. The mutually orthogonal spinors $\chi^\sigma(\varphi)$ can be expressed in terms of the eigenvectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, of the Pauli matrix σ_k, as

$$\chi^{\sigma=+}(\varphi) = \frac{1}{\sqrt{2\pi}}\begin{pmatrix} \cos\theta/2 \\ e^{i\varphi}\sin\theta/2 \end{pmatrix} \tag{7}$$

,

$$\chi^{\sigma=-}(\varphi) = \frac{1}{\sqrt{2\pi}}\begin{pmatrix} \sin\theta/2 \\ -e^{i\varphi}\cos\theta/2 \end{pmatrix} \tag{8}$$

.

B. Device geometry, boundary conditions and scattering matrix

The ring connected to the two leads and interrupted by the tunnel barrier in lower arm is shown in Fig.1. If the ring is not connected to external leads the boundary condition on the wave function implies that it has to be single valued when the argument φ is increased of 2π, which gives integer quantum numbers. When the ring is connected to external leads the boundary conditions are altered. In this case it is appropriate to apply the spin-dependent version of the Griffith boundary's condition[33] at the intersection. This reduces the description of the electron transport to a one-dimensional scattering problem. These boundary conditions state that (i) the wave function must be continuous and (ii) the spin density must be conserved. The same conditions on the continuity of the wave function and on the density conservation apply at the position of the tunnel barrier in the lower arm. Below, we briefly outline the determination of transmission and reflection coefficients of the scattering matrix problem (details can be found, e.g. in Ref.29).

Assumed that electrons in the two leads are free and have momentum k, the corresponding energy is $\hbar^2 k^2/2m^\star$. When an electron moves along the upper arm in the clockwise direction from the input intersection at $\varphi = 0$ (see Fig.1), it acquires a phase $\Phi^\sigma/2$ at the output intersection $\varphi = \pi$, whereas the electron acquires a phase $-\Phi^\sigma/4$ in the counterclockwise direction along the other arm when moving from $\varphi' = 0$ to $\varphi' = \pi/2$ and from $\varphi' = \pi/2$ to $\varphi' = \pi$, respectively. Therefore the total phase is Φ^σ when the electron goes through the loop. The electric field in the ring changes the momenta of the electrons in different spin states χ^σ as $k_+^\sigma = k + \Phi^\sigma/2\pi R$ and $k_-^\sigma = k - \Phi^\sigma/2\pi R$, where the subscript $\pm$ denotes the chirality. The wave functions in the upper(u) and lower(d) arm of the ring can be written as:

$$\Psi_u(\varphi) = \sum_{\sigma=\pm,\lambda=\pm} c_{u,\sigma}^\lambda e^{in_\lambda^\sigma \varphi} \chi^\sigma(\varphi),$$

$$\Psi_{d\alpha}(\varphi') = \sum_{\sigma=\pm,\lambda=\pm} c_{d\alpha,\sigma}^\lambda e^{-in_\lambda^\sigma \varphi'} \chi^\sigma(\varphi'), \tag{9}$$

where the index $d\alpha = d1, d2$ denotes the wave function in the two-halves of the lower branch and $n_\lambda^\sigma = \lambda k R + \Phi^\sigma/2\pi$. The wave function of the electron incident from the left lead in the left and right electrodes can be expanded as:

$$\Psi_L(x) = \Psi_i + (r_\uparrow, r_\downarrow)^T e^{-ikx}, \ \ \Psi_R(x) = (t_\uparrow, t_\downarrow)^T e^{ikx}, \tag{10}$$

where $x = R\varphi$, r_σ and t_σ are the spin-dependent reflection and transmission coefficient, Ψ_i is the wave function of the injected electron $\Psi_i = e^{ikx}\chi^\sigma(0)$. For an incident electron from the right lead an analogous expansion in terms of reflection and transmission coefficients is possible with i_σ, r_σ (for left lead) and $t_\sigma, 0$ (for right lead) replaced by $0, t_\sigma'$ and r_σ', i_σ'. This enables us to formulate the scattering matrix equation of the ring system as $\hat{o} = \hat{S}\hat{i}$, where $\hat{o}, \hat{i}$ stand for outgoing and incoming wave coefficients. In particular, the following relation holds: $t_\sigma = \sum_{\sigma'} T_{\sigma\sigma'} i_{\sigma'}$, $r_\sigma = \sum_{\sigma'} R_{\sigma\sigma'} i_{\sigma'}$ and a similar one for t_σ', r_σ'. Note that since the transmission amplitude does not depend on the choice of spinor basis, i.e., it is invariant under spin rotation, hereafter we make use of the ring-spinor basis where the spin flip amplitudes vanish, and $T_{\sigma,\sigma'}$ is diagonal, thus we define $T_{\sigma,\sigma} = T_\sigma$ and similarly for R_σ. The previous expression for wave functions are now used to calculate the transmission amplitude for the ring system from the proper requirements on wave function continuity and probability current conservation. The first Griffith boundary condition[33] states that the wave function is continuous at $\varphi = 0, \varphi' = 0$ and $\varphi = \pi, \varphi' = \pi$. Concerning the second boundary condition, if one assumes that there are no spin-flip-processes at the junctions, it requires that the spin current $J^\sigma = \text{Re}[(\Psi^\sigma \chi^\sigma)^\dagger \left(-i\frac{\partial}{\partial\varphi} + \frac{\beta}{2}\sigma_r - \frac{\Phi_{AB}}{\phi_0} \right) (\Psi^\sigma \chi^\sigma)]$ for each spin direction should be conserved, i.e. $J_u^\sigma + J_d^\sigma + J_{L(R)}^\sigma = 0$. At the point $\varphi' = \pi/2$ in the lower branch the same boundary conditions apply in presence of the delta tunnel barrier. Thus the system of equations to be solved is the following:

$$1 + r_\sigma = c_{u,\sigma}^+ + c_{u,\sigma}^- e^{-i\frac{\Phi^\sigma}{2}} = c_{d2,\sigma}^+ + c_{d2,\sigma}^- e^{i\frac{\Phi^\sigma}{4}} \tag{11}$$

$$c_{d2,\sigma}^+ e^{i\frac{\phi-\Phi^\sigma}{4}} + c_{d2,\sigma}^- e^{-i\frac{\phi}{4}} = c_{d1,\sigma}^+ e^{i\frac{\phi}{4}} + c_{d1,\sigma}^- e^{i\frac{\Phi^\sigma-\phi}{4}} \tag{12}$$

$$c_{u,\sigma}^+ e^{i\frac{\phi+\Phi^\sigma}{2}} + c_{u,\sigma}^- e^{-i\frac{\phi}{2}} = c_{d1,\sigma}^+ e^{i\frac{2\phi-\Phi^\sigma}{4}} + c_{d1,\sigma}^- e^{-i\frac{\phi}{2}} = t_\sigma e^{i\frac{\phi}{2}} \tag{13}$$

$$1 - r_\sigma = c_{u,\sigma}^+ - c_{u,\sigma}^- e^{-i\frac{\Phi^\sigma}{2}} + c_{d2,\sigma}^+ - c_{d2,\sigma}^- e^{i\frac{\Phi^\sigma}{4}} \tag{14}$$

$$c_{d2,\sigma}^+ e^{i\frac{\phi-\Phi^\sigma}{4}} - c_{d2,\sigma}^- e^{-i\frac{\phi}{4}} - c_{d1,\sigma}^+ e^{i\frac{\phi}{4}} + c_{d1,\sigma}^- e^{i\frac{\Phi^\sigma-\phi}{4}} = -iz(c_{d1,\sigma}^+ e^{i\frac{\phi}{4}} + c_{d1,\sigma}^- e^{i\frac{\Phi^\sigma-\phi}{4}}) \tag{15}$$

$$c_{u,\sigma}^+ e^{i\frac{\phi+\Phi^\sigma}{2}} - c_{u,\sigma}^- e^{-i\frac{\phi}{2}} + c_{d1,\sigma}^+ e^{i\frac{2\phi-\Phi^\sigma}{4}} - c_{d1,\sigma}^- e^{-i\frac{\phi}{2}} = t_\sigma e^{i\frac{\phi}{2}}, \tag{16}$$

where we have introduced $\phi = 2\pi k R = 2kL$. In the limit of zero tunnel barrier in the lower branch the above equations reduce to those in Ref.29. After some algebra we obtain the transmission coefficients $t_\sigma(\phi, \Phi^\sigma, z)$ and $t'_\sigma(\phi, \Phi^\sigma, z)$ where $z = v/k$.

The explicit expression of transmission coefficient $t_\sigma(\phi, \Phi^\sigma, z)$, written in a compact form, is the following:

$$t_\sigma(\phi, \Phi^\sigma, z) = \frac{8 \sin(\frac{\phi}{4}) \left(-4 \cos(\frac{\phi}{4}) \cos(\frac{\Phi^\sigma}{2}) + z \sin(\frac{\phi}{4}) e^{i\frac{\Phi^\sigma}{2}} \right)}{4 z \cos(\frac{\phi}{2}) - 2 (5 i + 2 z) \cos(\phi) + i \left(2 + 8 \cos(\Phi^\sigma) - 2 z \sin(\frac{\phi}{2}) + (8 i + 5 z) \sin(\phi) \right)}. \tag{17}$$

Its expression is determined by the tunnel barrier strength, the total phase, the kinetic state of the incident electrons, the electric field and the magnetic flux. In the limit $z \to 0$, it reduces to[29]:

$$t_\sigma(\phi, \Phi^\sigma) = \frac{8i \cos(\frac{\Phi^\sigma}{2}) \sin(\frac{\phi}{2})}{1 - 5 \cos(\phi) + 4 \cos(\Phi^\sigma) + 4 i \sin(\phi)}. \tag{18}$$

The conductance in the mesoscopic structure under consideration can be expressed by means of the Landauer-Büttiker conductance formula[34,35], which in our case reads:

$$G = (e^2/h) \sum_{\sigma=\uparrow,\downarrow} |T_\sigma|^2 = G_\uparrow + G_\downarrow, \tag{19}$$

where T_σ is the (spin dependent) transmission amplitude[37] introduced above, and G_σ is the spin-resolved conductance.

Already at this point one can envisage an application of the device as a spin filter. Assuming one can tune the phases Φ_{AB} and Φ_{AC} (via the magnetic field and the Rashba strength β) independently, and varying the tunnel barrier strength, one can make the ring almost transparent with high transmission probability only for electrons with spin quantum number $\sigma = \uparrow$ ($\downarrow$) and totally opaque to the electrons with $\sigma = \downarrow$ ($\uparrow$) by requiring $|T_\downarrow(\uparrow)|^2 = 0$ and $|T_\uparrow(\downarrow)|^2 \neq 0$. This effect will be discussed in details in next sections where we will focus on the effect of the tunnel barrier on spin-polarized transport.

III. TRANSMISSION AMPLITUDE FROM ONE-ELECTRON SCATTERING FORMALISM

A. General features

In quasi-one-dimensional (1D) systems, real conductance zeros appear under the condition of conserved time reversal symmetry[38,39] (TRS). The (anti)resonances in the transmission due to local quasibound states correspond to a specific zero-pole structure in the complex energy plane[40,41,42,43]. The application of an external magnetic field modifies this zero-pole structure, shifting the transmission zeros away from the real axis, with the shift as a function of the AB phase[44]. Thus, the lifting of zeros is related to the breaking of TRS. In analogy with the time-reversal symmetry breaking in AB ring, the lifting of the real conductance zeros in an AC ring can be related to spin-parity symmetry breaking as shown in Ref.11. In the following we discuss the effect of both TRS and spin-parity breaking, together with a rotational symmetry breaking in the presence of a tunnel barrier in the lower arm of the ring and will describe the novel fingerprints associated to the tunable barrier.

1. zero tunnel barrier

In absence of the tunnel barrier for SOC $\beta = 0$ and zero magnetic flux Φ_{AB}, the transmission function in Eq.(17) displays a peculiar resonant behavior as shown in Fig.2. The oscillation in the conductance for $z = 0$ is due to imperfect transmission caused by the coupling of the ring and the leads and therefore leads to resonances as a consequence of backscattering effects[10]. In the case $\beta \neq 0$ and for non-zero magnetic flux, i.e. for total flux $\Phi^\sigma \neq 0$, the transmission zeros are obtained from Eq.(18) as the solution of the equation:

$$\sin(\frac{\phi}{2}) \cos(\frac{\Phi^\sigma}{2}) = 0, \tag{20}$$

i.e. $\phi_{0,1} = 2n\pi$, $n \in \mathbb{Z}$. Such zeros correspond to the interference condition at the nodes. Besides, when $\phi/2\pi$ is not an integer, zeros can be obtained for $\Phi^\sigma = (2n+1)\pi$, $n \in \mathbb{Z}$.

2. non-zero tunnel barrier

When $z \neq 0$, and in presence of a net total flux, two types of zero appear. The zeros $\phi_{0,1}$, and the zeros $\phi_{0,2}$ which are determined by the geometry dependent interference condition at the tunnel barrier. From Eq.(17) the structural zeros determined by the tunnel barrier are given by the solution of the equation:

$$z \tan(\frac{\phi}{4})e^{i\frac{\Phi^\sigma}{2}} = 4 \cos \frac{\Phi^\sigma}{2}. \tag{21}$$

This equation, which is satisfied by values of the total flux $\Phi^\sigma = 2n\pi, n \in \mathbb{Z}$, gives the zeros of second type:

$$\frac{\phi_{0,2}}{2} = (2n+1)\pi - 2 \arctan(\frac{z}{4}). \tag{22}$$

The results for the spin-dependent transmission probability zeros are summarized in Fig.2. In absence of external electromagnetic fluxes periodical transmission zeros start to appear when $z \neq 0$, as discussed above.

3. conductance zeros and transmission resonances

By examination of the transmission amplitude in the complex energy plane we find a certain relation between the conductance zeros and the transmission resonances. To examine this connection, it is convenient to analyze the transmission amplitude in the complex energy plane by making the substitution $k \to k_R + ik_I$ and defining $x = e^{-k_I L}e^{ik_R L}$. In terms of the variable x the transmission amplitude can be rewritten as:

$$T_\sigma(x, \Phi^\sigma, z) = \frac{-4e^{i\frac{\Phi^\sigma}{2}}(x-1)\{x[(z-2i)e^{i\Phi^\sigma} - 2i] - (z+2i)e^{i\Phi^\sigma} - 2i\}}{x^4(z-2i)e^{i\Phi^\sigma} + x^3(2ze^{i\Phi^\sigma}) + 2ix^2(4e^{i2\Phi^\sigma} + 2e^{i\Phi^\sigma} + 4) + x(6ze^{i\Phi^\sigma}) - 9(z+2i)e^{i\Phi^\sigma}}. \tag{23}$$

The real zeros conductance are given by points belonging to the unitary circumference $|x|^2 = 1$ (which correspond to $k_I = 0$), whereas the poles have a finite imaginary part and correspond to points away from the unitary circumference. The poles correspond to maximum in the conductance curve, or to resonances (Fano types)[36] when k_I is finite but small. To find the real zeros conductance, we first solve the numerator w.r.t. x and then impose the condition $|x|^2 = 1$. As in the analysis above, when z is nonzero we can distinguish two types of zeros: Those corresponding to the interference condition at the nodes for $x = 1$, i.e. $\phi_{0,1} = 2k_R L = 2\pi n$ (n even) and those determined by the scattering at the tunnel barrier for:

$$x = \frac{ze^{i\Phi^\sigma} + 2i(1 + e^{i\Phi^\sigma})}{ze^{i\Phi^\sigma} - 2i(1 + e^{i\Phi^\sigma})}. \tag{24}$$

Imposing the condition $|x|^2 = 1$, which is satisfied by values of the total flux $\Phi^\sigma = 2n\pi, n \in \mathbb{Z}$, the zeros of second type are those given by Eq.(22). Their expression implies that for integer total flux the position of the real zeros conductance on the unitary circumference can be controlled by z.

To analyze the pole of the transmission amplitude, we need to solve a fourth order algebraic equation

$$\mathcal{D}(x) = x^4(z-2i)e^{i\Phi^\sigma} + x^3(2ze^{i\Phi^\sigma}) + 2ix^2(4e^{i2\Phi^\sigma} + 2e^{i\Phi^\sigma} + 4) + x(6ze^{i\Phi^\sigma}) - 9(z+2i)e^{i\Phi^\sigma}. \tag{25}$$

In the limit $z \to \infty$, $\mathcal{D}(x) \sim ze^{i\Phi^\sigma}(x-1)(x+3)(x-\sqrt{3}i)(x+\sqrt{3}i)$. The pole position is independent from the spin and the transmission amplitude is identical in both spin channels. In particular, the pole $x = -3$, which corresponds to $k_R L = \pi$, has a large imaginary part and gives rise to a large width of the resonance. The effect of z and of the total effective flux on the formation of zero-resonances in the spin-resolved conductance is shown in Figs.3-4 and in the complex energy plane in Fig.5. Note that when $z \neq 0$ structural zeros due to the scattering at the tunnel barrier start to appear (e.g. see down panel Fig.3). The last give rise to nonsymmetric maxima in the conductance.

B. spin filtering without magnets

Hereafter we show that the tunable barrier in one arm of the ring remarkably permits the AB ring in presence of SO interaction to operate as a spin-selective device. As discussed above, resonances (or poles) in the transmission amplitude do not necessarily give rise to zeros in the conductance. This is the case when zeros and poles compensate

each other and yield a finite value of the conductance. This property can be used to have a finite conductance in one spin-channel while being zero in the other channel. By examination of the transmission amplitude in the complex energy plane, we find that the conductance zeros and transmission resonances can be controlled by the height of the barrier z and the value of the total effective flux Φ^σ. In particular, by Eq.(25), we obtain that the zeros of first kind $\phi_{0,1}$ in one spin-channel ($x = 1$) can be compensated by imposing the condition on the denominator of the transmission amplitude $\mathcal{D}(x_p = 1) = 0$, while the zeros of second kind $\phi_{0,2}$ can be controlled by z. The equation $\mathcal{D}(x_p = 1) = 0$ can be rewritten as $(e^{i\Phi^\sigma} - 1)^2 = 0$, and is solved for integer values of the effective flux, $\Phi^\sigma = 2\pi n, n \in \mathbb{Z}$. Such solutions do not depend on z, on the contrary the location of zeros of second type can be controlled by z and correspond to energies:

$$\bar{k}L = (2n + 1)\pi - 2\arctan(\frac{z}{4}), \tag{26}$$

where $\bar{k}$ is the momentum of the electron for a given spin channel with vanishing transmission amplitude. The possibility of compensating the zeros of first kind in a selected spin channel and controlling of the location of zeros of second type by varying z points to the use of the AB-AC ring as a spin-filter. The spin-filtering effect is shown in Figs.6 and 7 for the choice $\beta = 1.2$ and $\bar{k}L = \frac{\pi}{2}, \frac{\pi}{4}$, respectively, while the other parameters are fixed according to Eq.(26). As shown in Fig.6 at $\bar{k}L = \pi/2 + 2n\pi$ the transmission probability of electrons with spin down $G_\downarrow$ is zero while $G_\uparrow$ is non-zero at $\bar{k}L$. It is worth to note that in the procedure above we have compensated the zeros of first kind only in one spin channel. If we would like to compensate a zero of the conductance in both spin channels we need to fix the values of the magnetic flux at $\Phi_{AB} = (2n + 1)\pi - \sigma\pi\sqrt{1 + \beta^2}$, being $n \in \mathbb{Z}$, which is satisfied by the values of $\beta = \sqrt{4m^2 - 1}$, $m \in \mathbb{Z}$. In this case we have a switching-effect controlled by z in both spin-channels.

To elucidate our spin-filter design we may turn to momentum resolved tunneling properties. It is well known that spin-orbit or Rashba interaction creates spin-orbit splitting of the conduction electron band in the ring. The variation of the tunnel barrier strength induces crossing points in the momentum dispersion curve of the external leads and of the Rashba-spin-split dispersion curve of the ring, simultaneously allowing tunneling for, e.g. spin-up right and left-movers at one node, while tunneling of spin-down electron is suppressed at the same node. Then electrons ending up in opposite external leads will have opposite spins. The tunnel barrier determines the location of the crossing points in the energy spectrum and thus acts as a spin filter. Equivalently, we may say that the transmission probability in the spin channel opposite to the incident spin orientation is the result of spin precession along the ring branches due to SOC as considered in Ref.45. The conductance zeros in the opposite channel correspond to a frequency of precession which reproduces the incident spin orientation at the junction. In our case the frequency precession is a function of the total flux enclosed in the ring and of the tunnel barrier strength.

In the context of spintronics[1,2] nonmagnetic spin-filters, as the one discussed here, are intriguing and, perhaps, useful alternative.

C. Temperature effects on the spin-filtering

For the spin-filter realization we are proposing it's relevant to evaluate the efficiency of the device at non-zero temperature. Thus in the following we generalize our calculations at finite temperature T. The conductance at finite T is given by

$$G = -(e^2/h) \sum_\sigma \int_0^\infty dE \frac{\partial f(E, \mu, T)}{\partial E} |T_\sigma(E, \Phi^\sigma, z)|^2, \tag{27}$$

where f is the Fermi distribution function, μ is the chemical potential and T the temperature. By transforming the integral over energy to an integral over the momentum k, and introducing the dimensionless variable $\xi = k/k_F$ (k_F being the Fermi momentum) we can express G as:

$$G(T) = (e^2/h) \sum_\sigma \int_0^\infty d\xi \xi \frac{T_F}{2T} \cosh^{-2}\left(\frac{T_F}{2T}(\xi^2 - \tilde{\mu})\right) |T_\sigma(2k_F\xi L, \Phi^\sigma, z)|^2, \tag{28}$$

where $\tilde{\mu}$ is the (dimensionless) chemical potential in units of Fermi energy E_F and T_F is the Fermi temperature. As discussed in Ref.29, for a InAs ring radius of $R = 0.25\mu m$ and effective mass $m^\star = 0.023$, the corresponding Fermi temperature is $T_F = 129.27$ K and $k_F = 20.5/R$.

In Fig.8 we report the results of the spin-resolved conductance for the following choice of parameters: $\beta = 1.2$, $\overline{k_F L} = 20.5\pi$, $z = 4\tan(\frac{\pi}{2} - \frac{\overline{k_F L}}{2})$, $\frac{\Phi_{AB}}{2\pi} = \frac{1}{2} + \frac{\sqrt{1+\beta^2}}{2}$ and three different temperatures: $T/T_F = 0.0008, 0.0010, 0.0012$.

As shown, the location of the zero in the down-spin transmission probability is not modified by the temperature. At the zero in the transmission probability of the down spin-channel corresponds a temperature dependent transmission probability in the up spin-channel when z meets the condition for spin-filtering Eq.(26) at fixed $\overline{k}_F$. The value of the transmission probability in the up spin-channel is reduced by fifty per cent when T/T_F increases from 0.0008 to 0.0012, i.e. for temperatures of the order $100mK$. Thus at the temperatures at which real devices are working the spin-filtering effect is still sizeable and the efficiency remains larger than fifty per cent.

IV. CONCLUSIONS

In summary, we have analyzed the zero conductance resonances appearing in an AB-AC ring as a signature of interfering resonant states of the loop system under the influence of a magnetic flux and of a Rashba electric field in presence of a tunable tunnel barrier. We have obtained, both numerically and analytically, the explicit dependence of the transmission on the spin-orbit coupling, the electromagnetic flux and the tunnel barrier, elucidating the role of internal and geometrical symmetries breaking in quantum transport and possible experimental realizations. According to Eq.(24) real zeros conductance are lifted by the influence of the external fields, being shifted into the complex plane depending on the value of the total flux phase and of the tunnel barrier strength. In the case of magnetic flux (spin-orbit interaction) is the breaking of time-reversal-symmetry (spin reversal symmetry) that destroys the energetic degeneracy of states preventing the destruction of interference effect at the nodes and leading to zeros. In the presence of a local tunnel barrier is the breaking of rotational symmetry to lead to zeros. More importantly, we have demonstrated how the presence of a tunable tunnel barrier can be used to control the polarization of transmitted electrons. We have shown a novel spin-filtering effect consisting in the possibility of compensating the interference zeros of the transmission probability in one spin-channel and controlling the location of the structural zeros by tuning the tunnel barrier strength. This effect points, once again, towards the possibility of employing one-dimensional rings as a spin-selective devices widening the field of usual magneto-electronics. We have also discussed the spin-filtering effect at finite temperature and shown that it remains sizeable when the temperature is raised up to 100 mK. We would like to stress that our proposal is within reach with today's technology for experiments in semiconductor heterostructures with a two dimensional electron gas (2DEG), e.g. InGaAs-based 2DEG, which has an internal electric field due to an asymmetric quantum well[31,46]. Indeed, spin interference effects in Rashba-gate-controlled ring embedded in the p-type self-assembled silicon quantum well with a quantum point contact inserted have recently been reported[47]. The proposed spin-filter device differs by previous proposals where the polarization of transmitted spin-polarized electrons can be controlled via an additional inhomogeneous magnetic field[22] or a symmetrically textured electric field[28]. Compared to these proposals, ours does not require additional fields and is related to a more controllable parameter, the tunnel barrier. We would like to stress that the presented results are valid for a one-dimensional ring, but they can be extended to rings of finite width w provided the inequality $w \ll R$ holds[29].

The question wether the spin-filtering effect survives when considering disordered-averaged quantities remains a further interesting problem that we are taking under consideration. Another interesting question is related to the presence of Dresselhaus spin-orbit interaction[27,48,49] that could lead to a similar conductance modulation and spin-filtering effect.

V. ACKNOWLEDGMENTS

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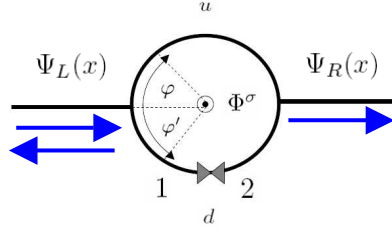[49] R. de Sousa and S. Das Sarma, Phys. Rev. B **68**, 155330 (2003).

FIG. 1: The one-dimensional ring symmetrically coupled to conducting leads and interrupted by a tunnel barrier in the lower arm.
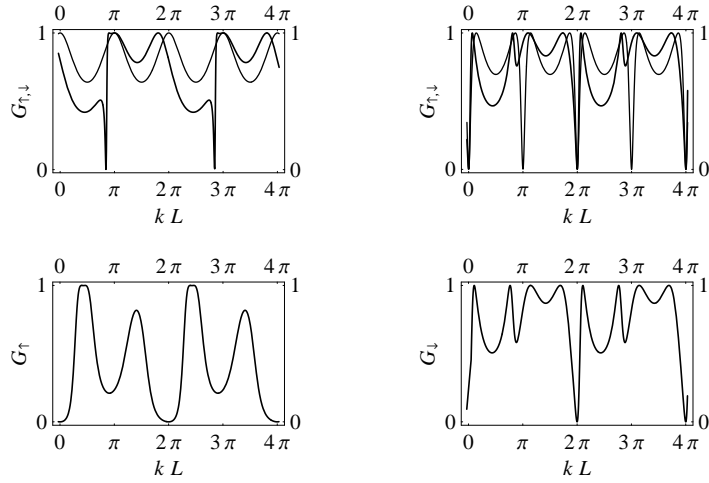

FIG. 2: The spin-resolved conductance G_σ as a function of kL. Left upper panel: $\frac{\Phi^\sigma}{2\pi} = 0$, $z = 0$ (thin line) and $z = 1$ (thick line). Right upper panel: $\frac{\Phi^\sigma}{2\pi} = 0.12$ (with $\frac{\Phi_{AB}}{2\pi} = 0.12, \beta = 0$), $z = 0$ (thin line) and $z = 1$ (thick line). When $z \neq 0$ periodic transmission zeros appear on top of the interference zeros at the nodes. Down panels: $\frac{\Phi_{AB}}{2\pi} = 0.12, \beta = 1.2$, $z = 1$. In the presence of SOC the two spin channels are distinct and real zeros are lifted.

FIG. 3: The spin resolved conductance G_σ versus kL for different values of the Aharonov-Bohm flux $\frac{\Phi_{AB}}{2\pi} = 0.04 \times n$, where $n = 0, 1, 2, 3, 4, 5$ from bottom to top and $z = 0$ (upper panels), $z = 0.6$ (lower panels). The SOC has been fixed at $\beta = 0$. In the upper panels only interference zeros appear ($kL = n\pi$), in the lower panel structural zeros conductance appear for $z \neq 0$ as discussed in the text.
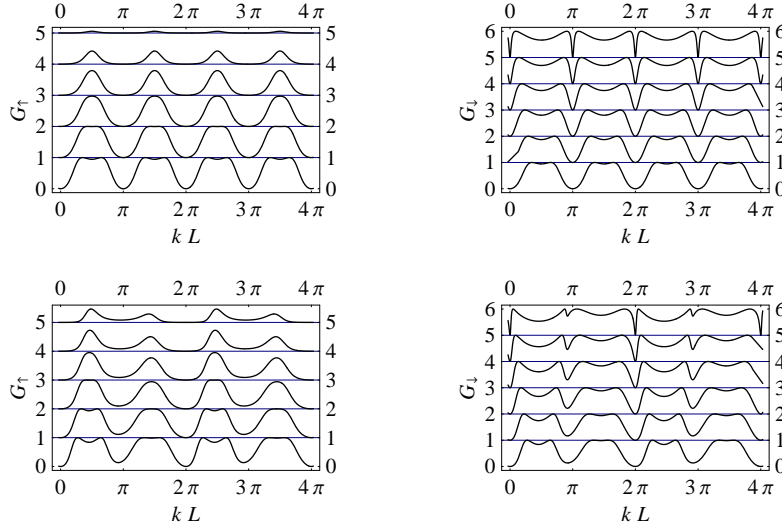

FIG. 4: The spin resolved conductance G_σ versus kL for different values of the Aharonov-Bohm flux $\frac{\Phi_{AB}}{2\pi} = 0.04 \times n$, where $n = 0, 1, 2, 3, 4, 5$ from bottom to top and $z = 0$ (upper panels), $z = 0.6$ (lower panels). The SOC has been fixed at $\beta = 1.2$. Due to the presence of the spin-orbit interaction spin-up and down channels have different zero-resonances structure.
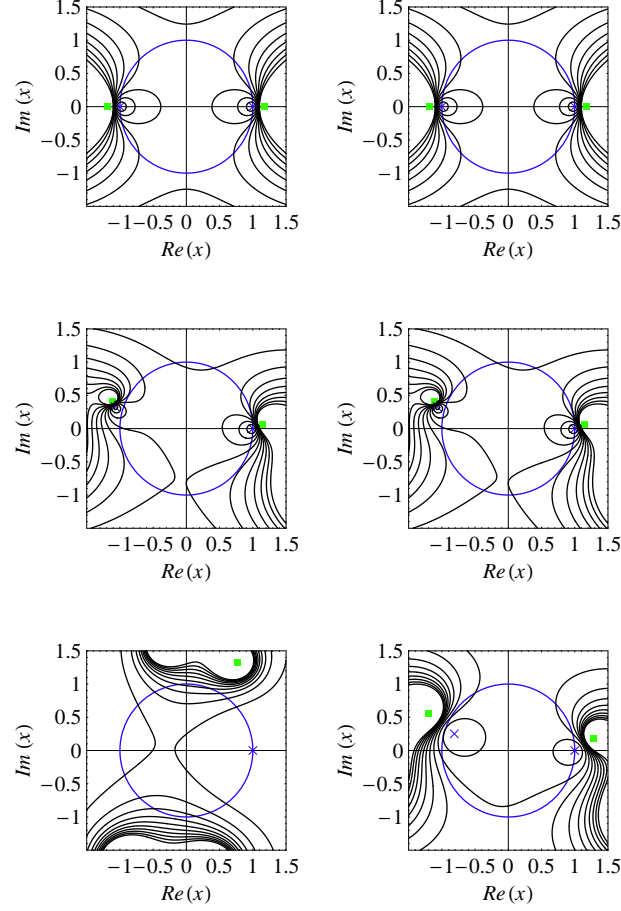
FIG. 5: Zeros (denoted by a cross) and poles (denoted by a box) in the complex energy plane for spin up (left panel) and spin down electrons (right panel) for the following choice of parameters: in the upper panels $z = 0$, $\frac{\Phi_{AB}}{2\pi} = 0.12$ $\beta = 0$ (corresponding to curve 3 of upper panel Figure 3); in the middle panels $z = 0.6$ $\frac{\Phi_{AB}}{2\pi} = 0.12$, $\beta = 0$ (corresponding to curve 3 of lower panel Figure 3); lower panel $z = 0.6$ $\frac{\Phi_{AB}}{2\pi} = 0.12$, $\beta = 1.2$ (corresponding to curve 3 of lower panel Figure 4). We show the contour plot of $|T_\sigma|^2$ and the unitary circle where real zeros conductance fall. Note that the location of the zeros and poles is on the real axis when the tunnel barrier is set to zero. The presence of a finite tunnel barrier lift the zero-pole structure from the real axis in the complex plane and gives rise to asymmetric resonance lineshape in the spin-resolved conductance (see Figure 3). The zero-pole structure is the same in both spin-channels in absence of spin-orbit interaction while spin-channels are distinct for β nonzero (lower panels).
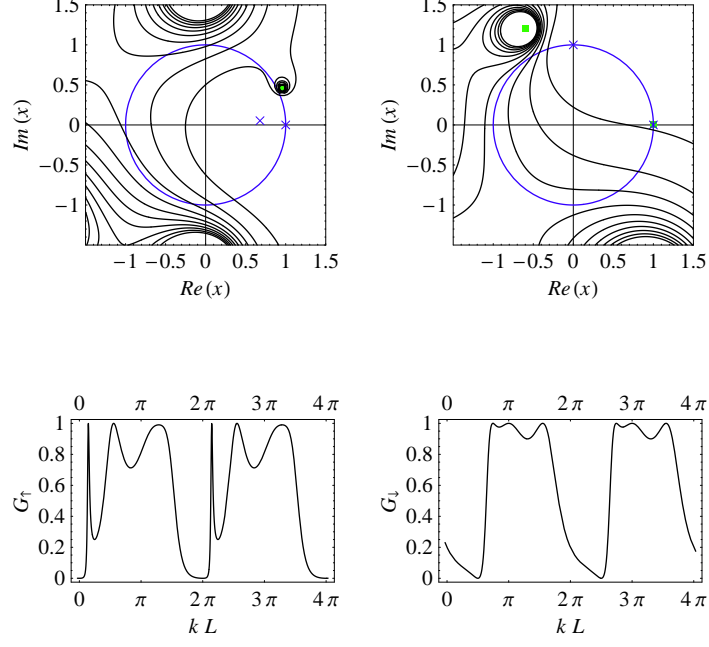
FIG. 6: Conductance curves (lower panels) and zero-pole structure (upper panels) for the following choice of parameters: $\beta = 1.2$, $\overline{k}L = \pi/2$, $z = 4\tan(\frac{\pi}{2} - \frac{\overline{k}L}{2})$, $\frac{\Phi_{AB}}{2\pi} = \frac{1}{2} + \frac{\sqrt{1+\beta^2}}{2}$. In the upper panels crosses denote the location of the zeros while the squares the location of the poles: The interference zeros in the down spin-channel are compensated by a pole at the same position, while the location of the zeros of second type on the unitary circumference is determined by the value of z. In the lower panels, at the zeros at $\overline{k}L = \pi/2 + 2n\pi$ in the down spin-channel correspond a finite conductance in the up spin-channel when z meets the condition for spin-filtering (26), as explained in the text. Sharp Fano-like resonances appear in the up spin channel due to the presence of a pole with a small imaginary part.
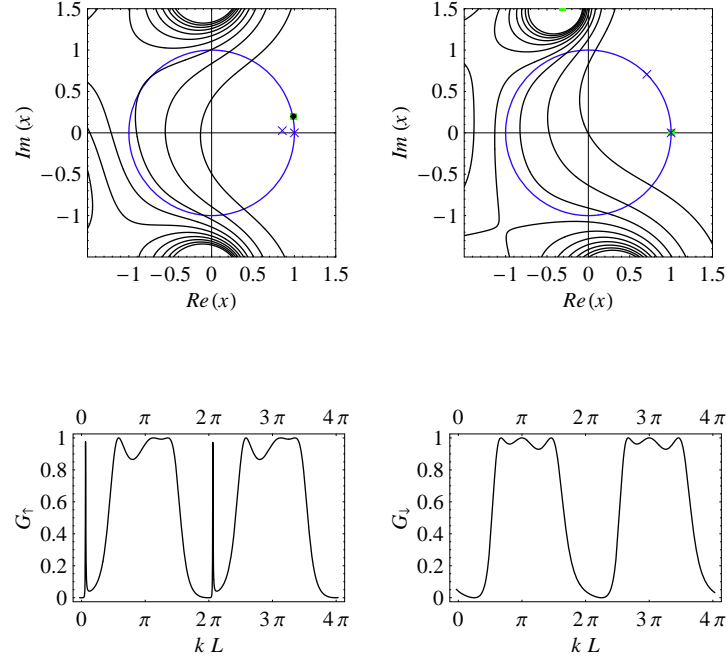
FIG. 7: The same as in Fig.6, with $\overline{k}L = \frac{\pi}{4}$.
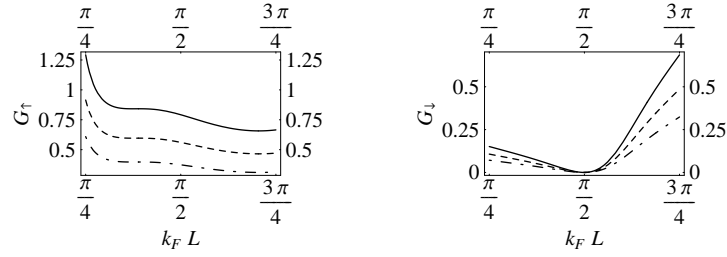

FIG. 8: Temperature dependence of spin filtering effect. The parameters used are the same as in Fig.6 for $T/T_F = 0.00083, 0.001, 0.00125$(from top to bottom) which correspond to temperatures of the order 100 mK in InGa. Only the behavior close to E_F ($k_F L = 20.5\pi$) has been plotted (all the values on the horizontal axis have been shifted by -20π).